Basic integration formulas (*u* is a function of *x*):

$\int kf(u)du = k\int f(u)du$

$\int [f(u) \pm g(u)]du = \int f(u)du \pm \int g(u)du$

$\int du = u + C$

$\int u^n du = \frac{u^{n+1}}{n+1} + C,\ n \in \mathbb{Q},\ \text{and } n \neq -1$

$\int \frac{1}{u}du = \int \frac{du}{u} = \int u^{-1}du = \ln|u| + C$

$\int e^u du = e^u + C$

$\int a^u du = \frac{a^u}{\ln a} + C$

$\int \sin u du = -\cos u + C$

$\int \cos u du = \sin u + C$

$\int \tan u du = -\ln|\cos u| + C$

$\int \cot u du = \ln|\sin u| + C$

$\int \sec u du = \ln|\sec u + \tan u| + C$

$\int \csc u du = -\ln|\csc u + \cot u| + C$ CORRECTED ON 1/25/18—COPY AND PASTE ERROR

$\int \sec^2 du = \tan u + C$

$\int \csc^2 du = -\cot u + C$

$\int \sec u \tan u du = \sec u + C$

$\int \csc u \cot u du = -\csc u + C$

$\int \frac{du}{\sqrt{a^2 - u^2}} = \arcsin\frac{u}{a} + C$

$$\int \frac{du}{u\sqrt{u^2-a^2}} = \operatorname{arcsec}\frac{|u|}{a} + C$$

ADDED ON 1/25/18 AND CORRECTED ON 1/30/18

$$\int \frac{du}{a^2+u^2} = \frac{1}{a}\arctan\frac{u}{a} + C$$

$$\int_a^b f(x)dx = F(b) - F(a)$$

where $f(x)$ is an antiderivative of $f(x)$.

ADDED ON 1/25/18

Algebra tools:

Rewrite an integrand having products of functions as sums and/or differences of functions

$$(a+b)^2 = a^2 + 2ab + b^2$$

$$(a-b)^2 = a^2 - 2ab + b^2$$

$$(a+b)(a-b) = a^2 - b^2$$

$$(a-b)(a^2+ab+b^2) = a^3 - b^3$$

$$(a+b)(a^2-ab+b^2) = a^3 + b^3$$

Rewrite an integrand with a rational function as sums and/or differences of functions

Simplify an integrand with a rational function by factoring

$$a^2 + 2ab + b^2 = (a+b)^2$$

$$a^2 - 2ab + b^2 = (a-b)^2$$

$$a^2 - b^2 = (a+b)(a-b)$$

$$a^3 - b^3 = (a-b)(a^2+ab+b^2)$$

$$a^3 + b^3 = (a+b)(a^2-ab+b^2)$$

Simplify rational function by polynomial long division

Rewrite the denominator of the integrand
by completing the square

$$x^2 + bx = \left[x^2 + bx + \left(\frac{b}{2}\right)^2 \right] - \left(\frac{b}{2}\right)^2$$

$$= \left(x + \frac{b}{2} \right)^2$$

Basic exponent rules

$$a^m a^n = a^{m+n} \qquad \frac{a^m}{a^n} = a^{m-n} \qquad a^m b^m = (ab)^m \qquad \frac{a^m}{b^m} = \frac{a^m}{b^m} \qquad a^{mn} = (a^m)^n = (a^n)^m \qquad \sqrt[n]{a^m} = a^{m/n}$$

Trigonometry tools:

Rewrite integrand using identities

Multiply numerator and denominator of
integrand by the Pythagorean conjugate

Integrating composite functions:

$$\int f(u)du = F(u) + C$$
$(u = g(x), \text{change of variables})$

$$\int_{u(a)}^{u(b)} f(u)du = F[u(b)] - F[u(a)]$$
$(\text{change limits of integration before integrating})$

$$\int_a^b f[g(x)]g'(x)dx = F(b) - F(a)$$
$(\text{sub. } u = g(x), \text{ then find } \int f(u)\text{du, then back sub } u = g(x) \text{ into } F(u))$

$$\int f[g(x)]g'(x)dx = F[g(x)] + C$$
$(\text{pattern recognition})$

$$\int_a^b f[g(x)]g'(x)dx = F(b) - F(a)$$
$(\text{use pattern recognition and no changes to the limits of integration needed})$